Limiting Lagrangian Formalism of Massive Spin-2 Particles

Mitsuru YAMADA*

(Received September 8, 1986)

Abstract — Quantum field theory of massive high-spin particles, usually thought unrenormalizable, is considered as a limit of renormalizable but nonunitary theory. General criterion such a theory must satisfy is presented for arbitrary spin. The limiting procedure for massive spin-2 particles is shown by explicitly constructing one-parameter Lagrangian and propagator.

The Lagrangian formalism of massive particles with spin larger than one half is a field theory with constraints. Besides independent canonical variables there are dependent variables which must be eliminated using the constraints. The presence of these variables makes the propagator ill-behaved in high-energy limit, invalidates the power counting rule for Feynman amplitudes and then leads to unrenormalizability of the whole theory. One way out of this impasse is to regard the usual Lagrangian as a limiting value of renormalizable Lagrangians. According to this idea there must be one-parameter family of Lagrangian $\mathscr{L}_\theta$ which, for finite values of the parameter θ, gives a renormalizable theory. This theory, however, is nonunitary because of many unwanted singularities of the propagator. If one takes the limit $\theta \to 0$ after renormalization is performed, all undesirable singularities must escape to infinity so that the theory should restore unitarity. Assuming that the renormalized result has a finite limit, we can expect that this is the correct theory. Lee-Yang's ξ-limiting procedure of charged vector boson[1] was the first example along this programme. Subsequently Munczek gave a similar formalism for the massive spin-$\frac{3}{2}$ field[2]. In this paper we extend this procedure to massive spin-2 field.

Before giving renormalizable Lagrangian and propagator for this case, let us consider what requirements must be satisfied for this formalism to be extended to a general case of spin S. Suppose that Lagrangian density $\mathscr{L}_0 = \bar{\psi}(x)\Lambda_0(i\partial)\psi(x)$ gives unitary but unrenormalizable theory of massive spin-S particles[3]. In this theory the free propagator $D_0(p)$ must be the inverse matrix of $\Lambda_0(p)$ so that $\Lambda_0(p)D_0(p) = D_0(p)\Lambda_0(p) = I$, where I is $N\times N$ unit matrix, N being the number of components of ψ field. We must find out one-parameter family of Lagrangian matrix $\Lambda(p,\theta)$ and propagator $D(p,\theta)$, which fits the following requirements:

(i) $\Lambda(p,\theta)D(p,\theta)=D(p,\theta)\Lambda(p,\theta)=I$, and $\Lambda(p,0)=\Lambda_0(p)$, $D(p,0)=D_0(p)$.

(ii) For finite fixed θ and large p^2, all N^2 elements of $D(p,\theta)$ behave as $O(1/p^2)$.

(iii) One can take the limit of $\theta\to 0$ in such a way that all singularities of $D(p,\theta)$ except the pole at $p^2 = m^2$ escape to infinity through *positive* real axis of complex p^2-plane.

The second requirement guarantees the power counting rule for Feynman amplitudes. Our primary concern here is bosons. For fermions the behaviour is replaced by $O(1/\sqrt{p^2})$. The third one is neccessary also for Wick rotation of the Feynman integral to be made in a usual way.

As massive spin-2 particles are described by a symmetric rank-2 tensor $\psi(x)_{\alpha\beta}$, Lagrangian density we are looking for is written as $\mathscr{L}_\theta = \bar{\psi}(x)_{\alpha\beta}\Lambda(i\partial,\theta)^{\alpha\beta\gamma\delta}\psi(x)_{\gamma\delta}$. Let us introduce following six matrices

* General Education, College of Technology, Ibaraki University, Hitachi 316, Japan

$$S_1^{\alpha\beta\gamma\delta}=\frac{1}{2}(g^{\alpha\gamma}g^{\beta\delta}+g^{\alpha\delta}g^{\beta\gamma}),$$

$$S_2^{\alpha\beta\gamma\delta}=\frac{1}{4}(p^\alpha g^{\beta\gamma}p^\delta+p^\alpha g^{\delta\beta}p^\gamma+p^\beta g^{\alpha\gamma}p^\delta+p^\beta g^{\alpha\delta}p^\gamma),$$

$$S_3^{\alpha\beta\gamma\delta}=g^{\alpha\beta}g^{\gamma\delta},$$

$$S_4^{\alpha\beta\gamma\delta}=g^{\alpha\beta}p^\gamma p^\delta,$$

$$S_5^{\alpha\beta\gamma\delta}=p^\alpha p^\beta g^{\gamma\delta},$$

$$S_6^{\alpha\beta\gamma\delta}=p^\alpha p^\beta p^\gamma p^\delta. \tag{1}$$

Every scalar matrix of our space is linearly dependent on them. The Lagrangian and the propagator of the unitary theory are given by [3,4]

$$\Lambda_0(p)=(p^2-m^2)(S_1-S_3)-2S_2+S_4+S_5, \tag{2}$$

$$D_0(p)=\left\{S_1-\frac{1}{3}S_3-2S_2/m^2+(S_4+S_5)/(3m^2)+2S_6/(3m^4)\right\}/(p^2-m^2), \tag{3}$$

while those of our limiting theory are given by

$$\Lambda(p,\theta)=\Lambda_0(p)+\theta(2S_2-p^2S_3-\frac{1}{2}S_4-\frac{1}{2}S_5), \tag{4}$$

$$D(p,\theta)=\Big\{S_1+2(1-\theta)S_2/(\theta p^2-m^2)-\frac{1}{3}S_3 \\ -\left(\frac{2}{3}-\theta\right)(S_4+S_5)/(3\theta p^2-2m^2)+KS_6\Big\}/(p^2-m^2), \tag{5}$$

$$K=\frac{2}{3}\left\{\theta(4-15\theta+9\theta^2)p^2-2(2-6\theta+3\theta^2)m^2\right\}/\left\{(\theta p^2-m^2)(3\theta p^2-2m^2)^2\right\}. \tag{6}$$

As long as θ approaches zero from positive side, this set of $\Lambda(p,\theta)$ and $D(p,\theta)$ satisfies requirements (i)-(iii). $D(p,\theta)$ has another representation by partial fractions:

$$D(p,\theta)=D_0(p)+(2S_2/m^2-2\theta S_6/m^4)/(p^2-m^2/\theta) \\ -\left\{(S_4+S_5)/(3m^2)+2(1-3\theta)S_6/(3m^4)\right\}/\left\{p^2-2m^2/(3\theta)\right\} \\ +\left\{4S_6/(9\theta m^2)\right\}/\left\{p^2-2m^2/(3\theta)\right\}^2. \tag{7}$$

Here the residue of the pole at $p^2=m^2/\theta$ is just the projection operator for spin-1 space. The singularity at $p^2=2m^2/(3\theta)$ results from the collision of two poles, each of which is a contribution from a spin-0 state. Thus the spectrum consists of particles of spin 2, mass m; spin 1, mass $m/\sqrt{\theta}$; and spin 0, mass $m\sqrt{2/(3\theta)}$. The spin-1 particle comes in negative metric and that of spin zero in dipole, but these unwanted particles disapear in the limit $\theta \to +0$.

For perturbative calculations of nonderivative interactions of ψ, we only have to use naive Feynman rules: i.e. put $H_{\text{int}} = -\mathscr{L}_{\text{int}}$ and use the propagator (5). This is another advantage of our limiting formalism. If we set $\theta = 0$ from the beginning and started with Lagrangian (2), the standard canonical quantization would not produce the naive Feynman rules. In fact, we must use an effective Hamiltonian which has an extra term proportional to $\delta^4(0)$[1,5].

Two final remarks: Firstly, quite similar to Lee-Yang's spin-1 case, whether the renormalized S-matrix elements of specific interactions have finite limit of $\theta \to +0$ requires separate investigations. Secondly, contrary to Lee-Yang's case, there can be many other possible procedures to approach the same $\mathscr{L}_0$ in spin-2 case. The above presented one is perhaps the simplest among them.

REFERENCES

[1] T.D. LEE and C.N. YANG, *Phys. Rev.* 128 (1962) 885.
[2] H. MUNCZEK, *Phys. Rev.* 164 (1967) 1794.
[3] Y. TAKAHASHI, *An Introduction to Field Quantization* (Pergamon, New York, 1969) Chap.3.
[4] L.M. NATH, *Nucl. Phys.* 68 (1965) 660; S.C. BHARGAVA and H. WATANABE, *Nucl. Phys.* 87 (1966) 273.
[3] S. WEINBERG, *Phys. Rev.* D7 (1973) 1068.